

SOME NEW DECOMPOSITION THEOREMS

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Abstract. In 1968, N. V. Velic'ko [11] introduced the concepts of δ -closed sets, δ -open sets, δ -closure and δ -interior operators in topological spaces. In [4], the concept of λ -closed sets was introduced. In this paper, α - δ w-open sets, pre- δ w-open sets, semi- δ w-open sets, strongly β - δ w-open sets and b- δ w-open sets in topological spaces are introduced and investigated. We introduce new classes of sets by using λ - δ w-closed sets in topological spaces and study their basic properties; and their connections with other types of topological sets. Furthermore, some new decomposition theorems are obtained.

1. Introduction and Preliminaries

By a space X , we always mean a topological space (X, τ) with no separation properties assumed. If $H \subseteq X$, $\text{cl}(H)$ and $\text{int}(H)$ will, respectively, denote the closure and interior of H in (X, τ) .

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Definition 1.1. [10] A subset H of a space X is said to be

(1) regular open if $H = \text{int}(\text{cl}(H))$. (2)

regular closed if $H = \text{cl}(\text{int}(H))$.

The complement of a regular open set is called regular closed.

Definition 1.2. A subset H of a space X is said to be

(1) α -open [9] if $H \subseteq \text{int}(\text{cl}(\text{int}(H)))$.

(2) preopen [8] if $H \subseteq \text{int}(\text{cl}(H))$.

(3) semi-open [6] if $H \subseteq \text{cl}(\text{int}(H))$.

(4) β -open [1] if $H \subseteq \text{cl}(\text{int}(\text{cl}(H)))$.

(5) b-open [3] if $H \subseteq \text{int}(\text{cl}(H)) \cup \text{cl}(\text{int}(H))$.

Definition 1.3. A subset A of a space X is called a Λ -set if $A = A^\Lambda$ where $A^\Lambda = \bigcap \{G : A \subseteq G \text{ and } G \in \tau\}$ [7].

Definition 1.4. A subset O of a space X is called λ -closed [4] if $O = L \cap M$, where L is a Λ -set and M is closed.

Proposition 1.5. [7] In a space X , every open set is a Λ -set but not conversely.

A point $x \in X$ is called a δ -cluster [11] of $H \subseteq X$ if $H \cap G \neq \emptyset$ for each regular open set G containing x .

The set of all δ -cluster points of H is called the δ -closure of H and is denoted by $\text{cl}_\delta(H)$.

A subset H of a space X is called δ -closed if $\text{cl}_\delta(H) = H$.

The complement of a δ -closed set is called δ -open.

The collection of all δ -open subsets of X forms a topology τ_δ on X . Indeed $\tau_\delta \subset \tau$. Let A be a subset of a space X . Then

(1) $\text{cl}_\delta(X - A) = X - \text{int}_\delta(A)$. (2)

$\text{int}_\delta(X - A) = X - \text{cl}_\delta(A)$.

Let us say that $w \subseteq \wp(X)$ is a weak structure (briefly WS) on X iff $\emptyset \in w$. Clearly each generalized topology and each minimal structure is a WS [5].

Each member of w is said to be w -open and the complement of a w -open set is called w -closed.

Let w be a weak structure on X and $H \subseteq X$. We define (as in the general case) $i_w(H)$ is the union of all w -open subsets contained in H and $c_w(H)$ is the intersection of all w -closed sets containing H [5].

Remark 1.6. [2] If w is a WS on X , then $i_w(\emptyset) = \emptyset$ and $c_w(X) = X$.

Theorem 1.7. [5] If w is a WS on X and $A, B \in w$ then

- (1) $i_w(A) \subseteq A \subseteq c_w(A)$,
- (2) $A \subseteq B \Rightarrow i_w(A) \subseteq i_w(B)$ and $c_w(A) \subseteq c_w(B)$, (3)
- $i_w(i_w(A)) = i_w(A)$ and $c_w(c_w(A)) = c_w(A)$,
- (4) $i_w(X - A) = X - c_w(A)$ and $c_w(X - A) = X - i_w(A)$.

Let w be a WS on X and $H \subseteq X$. Then $H \in \alpha(w)$ [resp. $H \in \sigma(w)$, $H \in \pi(w)$, $H \in \beta(w)$, $H \in b(w)$] if $H \subseteq i_w(c_w(i_w(H)))$ [resp. $H \subseteq c_w(i_w(H))$, $H \subseteq i_w(c_w(H))$, $H \subseteq c_w(i_w(c_w(H)))$, $H \subseteq i_w(c_w(H)) \cup c_w(i_w(H))$] [5].

2. Generalized δw -open sets

Definition 2.1. Let w be a weak structure on a space X . A subset G of X is called

- (1) pre- δw -open if $G \subseteq i_w(cl_\delta(G))$. (2)
- semi- δw -open if $G \subseteq cl_\delta(i_w(G))$. (3) α -
- δw -open if $G \subseteq i_w(cl_\delta(i_w(G)))$.
- (4) strongly β - δw -open if $G \subseteq cl_\delta(i_w(cl_\delta(G)))$. (5)
- b - δw -open if $G \subseteq i_w(cl_\delta(G)) \cup cl_\delta(i_w(G))$.

The complement of a pre- δw -open (resp. a semi- δw -open, an α - δw -open, a strongly β - δw -open, a b - δw -open) set is called pre- δw -closed (resp. semi- δw -closed, α - δw -closed, strongly β - δw -closed, b - δw -closed).

Definition 2.2. For a WS w on a space X and a subset G of X , the intersection of all pre- δw -closed (resp. semi- δw -closed, α - δw -closed, strongly β - δw -closed, b - δw -closed) sets containing G is called the pre- δw -closure (resp. semi- δw -closure, α - δw -closure, strongly β - δw -closure, b - δw -closure) of G and is denoted by $\text{pcl}_{\delta w}(G)$ (resp. $\text{scl}_{\delta w}(G)$, $\alpha\text{cl}_{\delta w}(G)$, $\beta\text{cl}_{\delta w}(G)$, $b\text{cl}_{\delta w}(G)$).

Proposition 2.3. For a WS w on a space X , we have

- (1) Every w -open set is α - δw -open.
- (2) Every α - δw -open is semi- δw -open. (3)
- Every α - δw -open set is pre- δw -open.
- (4) Every pre- δw -open set is strongly β - δw -open. (5)
- Every semi- δw -open set is strongly β - δw -open.

Proof. (1) Let H be a w -open set. Then $H = i_w(H)$. Since $H \subseteq \text{cl}_{\delta}(H)$, $i_w(H) \subseteq \text{cl}_{\delta}(i_w(H))$ and $i_w(H) \subseteq i_w(\text{cl}_{\delta}(i_w(H))) \Rightarrow H \subseteq i_w(\text{cl}_{\delta}(i_w(H)))$. This shows that H is an α - δw -open set.

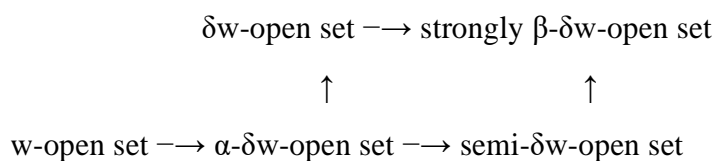
(2) Let H be an α - δw -open set. Then $H \subseteq i_w(\text{cl}_{\delta}(i_w(H))) \subseteq \text{cl}_{\delta}(i_w(H))$. This shows that H is a semi- δw -open set.

(3) Let H be an α - δw -open set. Then $H \subseteq i_w(\text{cl}_{\delta}(i_w(H))) \subseteq i_w(\text{cl}_{\delta}(H))$. This shows that H is a pre- δw -open set.

(4) Let H be a pre- δw -open set. Then $H \subseteq i_w(\text{cl}_{\delta}(H))$ and $\text{cl}_{\delta}(H) \subseteq \text{cl}_{\delta}(i_w(\text{cl}_{\delta}(H)))$. This shows that H is strongly β - δw -open.

(5) Let H be a semi- δw -open set. Then $H \subseteq \text{cl}_{\delta}(i_w(H)) \subseteq \text{cl}_{\delta}(i_w(\text{cl}_{\delta}(H)))$. This shows that H is a strongly β - δw -open.

Remark 2.4. For several subsets defined above, we have the following implications. pre-



The reverse implications are not true.

Example 2.5. Let $X=\{a, b, c, d\}$, $\tau=\{\emptyset, \{a\}, \{a, b, c\}, X\}$ and $w=\{\emptyset, \{b, c\}, \{a, c, d\}, X\}$.

Then $\{a, b, c\}$ is an α - δw -open set but it is not an w -open set.

Example 2.6. Let $X=\{a, b, c, d\}$, $\tau=\{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ and $w=\{\emptyset, \{a\}, \{d\}, \{a, b, c\}, X\}$. Then $\{b, d\}$ is a semi- δw -open set but it is not an α - δw -open set.

Example 2.7. In Example 2.6, $\{c\}$ is a pre- δw -open set but it is not an α - δw -open set.

Example 2.8. In Example 2.6, $\{b, d\}$ is a strongly β - δw -open set but it is not a pre- δw -open set.

Example 2.9. In Example 2.6, $\{c, d\}$ is a strongly β - δw -open set but it is not a semi- δw -open set.

Remark 2.10. For a WS on a space X and a subset A of X , we have

$$(1) \text{pcl}_{\delta w}(A) = A \cup c_w(\text{int}_{\delta}(A)). \quad (2)$$

$$\text{scl}_{\delta w}(A) = A \cup \text{int}_{\delta}(c_w(A)).$$

$$(3) \text{acl}_{\delta w}(A) = A \cup c_w(\text{int}_{\delta}(c_w(A))). \quad (4)$$

$$\text{s}\beta\text{cl}_{\delta w}(A) = A \cup \text{int}_{\delta}(c_w(\text{int}_{\delta}(A))). \quad (5)$$

$$\text{bcl}_{\delta w}(A) = \text{pcl}_{\delta w}(A) \cap \text{scl}_{\delta w}(A).$$

Definition 2.11. Let w be a WS on a space X . A subset A of X is called

- (1) a $\Lambda_{\alpha w}$ -set if $A = \Lambda_{\alpha w}(A)$ where $\Lambda_{\alpha w}(A) = \bigcap \{G : A \subseteq G \text{ and } G \in \alpha(w)\}$. (2) a Λ_{sw} -set if $A = \Lambda_{sw}(A)$ where $\Lambda_{sw}(A) = \bigcap \{G : A \subseteq G \text{ and } G \in \sigma(w)\}$. (3) a Λ_{pw} -set if $A = \Lambda_{pw}(A)$ where $\Lambda_{pw}(A) = \bigcap \{G : A \subseteq G \text{ and } G \in \pi(w)\}$. (4) a $\Lambda_{\beta w}$ -set if $A = \Lambda_{\beta w}(A)$ where $\Lambda_{\beta w}(A) = \bigcap \{G : A \subseteq G \text{ and } G \in \beta(w)\}$. (5) a Λ_{bw} -set if $A = \Lambda_{bw}(A)$ where $\Lambda_{bw}(A) = \bigcap \{G : A \subseteq G \text{ and } G \in b(w)\}$.

Definition 2.12. Let w be a WS on a space X . A subset K of X is called

- (1) a $C_{\delta w}$ -set if $K = L \cap F$, where L is open and F is a pre- δw -closed set in X . (2) a $BC_{\delta w}$ -set if $K = L \cap F$, where L is open and F is a b- δw -closed set in X . (3) an $\eta_{\delta w}$ -set if $K = L \cap F$, where L is open and F is an α - δw -closed set in X .

Definition 2.13. Let w be a WS on a space X . A subset K of X is called

- (1) λ - δw - α -closed if $K = L \cap F$, where L is a Λ -set and F is an α - δw -closed set. (2) a $DC_{\delta w}$ -set if $K = L \cap F$, where L is open and F is a semi- δw -closed set. (3) a $sFC_{\delta w}$ -set if $K = L \cap F$, where L is open and F is a strongly β - δw -closed set.

Lemma 2.14. Let w be a WS on a space X . Then the following statements hold. (1)

Every α - δw -closed set is λ - δw - α -closed.

- (2) Every Λ -set is λ - δw - α -closed.

Example 2.15. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $w = \{\emptyset, \{a, b\}, X\}$. Then

- (1) $\{a\}$ is λ - δw - α -closed but not α - δw -closed. (2) $\{c\}$ is λ - δw - α -closed but not a Λ -set.

Lemma 2.16. For a WS w on a space X and a subset A of X , then the following conditions are equivalent.

- (1) A is λ - δw - α -closed.
(2) $A = L \cap \alpha cl_{\delta w}(A)$ where L is a Λ -set. (3) $A = A^{\Lambda} \cap \alpha cl_{\delta w}(A)$.

Lemma 2.17. For a WS w on a space X , the following statements hold. (1)

- Every open set is an $\eta_{\delta w}$ -set.
(2) Every α - δw -closed set is an $\eta_{\delta w}$ -set.

Example 2.18. In Example 2.15,

- (1) $\{c\}$ is an $\eta_{\delta w}$ -set but not open.

(2) $\{a, b\}$ is an $\eta_{\delta w}$ -set but not an α - δw -closed.

Lemma 2.19. Let w be a WS on a space X . Then every $\eta_{\delta w}$ -set is λ - δw - α -closed.

Proposition 2.20. Let w be a WS on a space X . Then the concepts of

- (1) α -open sets and α - δw -open sets are independent.
- (2) semi-open sets and semi- δw -open sets are independent. (3)
- preopen sets and pre- δw -open sets are independent.
- (4) β -open sets and strongly β - δw -open sets are independent. (5)
- b -open sets and b - δw -open sets are independent.

Example 2.21. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $w = \{\emptyset, \{a\}, \{b, c\}\}$.

Then

- (1) $\{a\}$ is an α - δw -open set but it is not an α -open set. (2)
- $\{b\}$ is an α -open set but it is not an α - δw -open set.

Example 2.22. In Example 2.21,

- (1) $\{a\}$ is a semi- δw -open set but it is not a semi-open set. (2)
- $\{b\}$ is a semi-open set but it is not a semi- δw -open set.

Example 2.23. In Example 2.21,

- (1) $\{a\}$ is a pre- δw -open set but it is not a preopen set. (2)
- $\{b\}$ is a preopen set but it is not a pre- δw -open set.

Example 2.24. In Example 2.21,

- (1) $\{a\}$ is a strongly β - δw -open set but it is not a β -open set. (2)
- $\{b\}$ is a β -open set but it is not a strongly β - δw -open set.

Example 2.25. In Example 2.21,

- (1) $\{a\}$ is a b - δw -open set but it is not a b -open set. (2)
- $\{b\}$ is a b -open set but it is not a b - δw -open set.

Definition 2.26. Let w be a WS on a space X . A subset A of X is said to be

- (1) δw_{ag} -closed if $\alpha cl_{\delta w}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. (2)
- δw_{gs} -closed if $scl_{\delta w}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. (3) δw_{gp} -
- closed if $pcl_{\delta w}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. (4) δw_{gsp} -
- closed if $s\beta cl_{\delta w}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. (5) δw_{gb} -
- closed if $bcl_{\delta w}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Lemma 2.27. Let w be a WS on a space X . A subset $A \subseteq X$ is δw_{ag} -closed if and only if $\alpha cl_{\delta w}(A) \subseteq A^\Delta$.

Theorem 2.28. For a WS w on a space X and a subset A of X , the following conditions are equivalent.

- (1) A is α - δw -closed.
- (2) A is δw_{ag} -closed and an $\eta_{\delta w}$ -set.
- (3) A is δw_{ag} -closed and λ - δw - α -closed.

Proof. (1) \Rightarrow (2) and (2) \Rightarrow (3) : Obvious.

(3) \Rightarrow (1) : Since A is δw_{ag} -closed, by Lemma 2.27, $\alpha cl_{\delta w}(A) \subseteq A^\Delta$. Since A is λ - δw - α -closed, by Lemma 2.16, $A = A^\Delta \cap \alpha cl_{\delta w}(A) = \alpha cl_{\delta w}(A)$. Hence A is α - δw -closed.

Remark 2.29. The following example shows that the concepts of δw_{ag} -closed sets and $\eta_{\delta w}$ -sets are independent of each other.

Example 2.30. (1) In Example 2.15, $\{a\}$ is an $\eta_{\delta w}$ -set but not δw_{ag} -closed. (2) In Example 2.15, $\{a, c\}$ is δw_{ag} -closed but not an $\eta_{\delta w}$ -set.

Remark 2.31. The following example shows that the concepts of δw_{ag} -closed sets and λ - δw - α -closed sets are independent of each other.

Example 2.32. (1) In Example 2.15, $\{a\}$ is λ - δw - α -closed but not δw_{ag} -closed. (2) In Example 2.15, $\{a, c\}$ is δw_{ag} -closed but not λ - δw - α -closed.

Definition 2.33. Let w be a WS on a space X . A subset K of X is called

- (1) λ - δw -semi-closed if $K = L \cap F$, where L is a Λ -set and F is semi- δw -closed.
- (2) λ - δw -pre-closed if $K = L \cap F$, where L is a Λ -set and F is pre- δw -closed
- (3) λ - δw - β -closed if $K = L \cap F$, where L is a Λ -set and F is strongly β - δw -closed.
- (4) λ - δw -b-closed if $K = L \cap F$, where L is a Λ -set and F is b- δw -closed.

Lemma 2.34. Let w be a WS on a space X . A subset $A \subseteq X$ is

- (1) δw_{gs} -closed if and only if $scl_{\delta w}(A) \subseteq A^\Lambda$.
- (2) δw_{gp} -closed if and only if $pcl_{\delta w}(A) \subseteq A^\Lambda$.
- (3) δw_{gsp} -closed if and only if $s\beta cl_{\delta w}(A) \subseteq A^\Lambda$.
- (4) δw_{gb} -closed if and only if $bcl_{\delta w}(A) \subseteq A^\Lambda$.

Corollary 2.35. For a WS w on a space X and a subset A of X , the following conditions are equivalent.

- (1) (i) A is semi- δw -closed.
(ii) A is δw_{gs} -closed and a $DC_{\delta w}$ -set.
(iii) A is δw_{gs} -closed and λ - δw -semi-closed.
- (2) (i) A is pre- δw -closed.
(ii) A is δw_{gp} -closed and a $C_{\delta w}$ -set.
(iii) A is δw_{gp} -closed and λ - δw -pre-closed.
- (3) (i) A is strongly β - δw -closed.
(ii) A is δw_{gsp} -closed and a $sFC_{\delta w}$ -set. (iii)
 A is δw_{gsp} -closed and λ - δw - β -closed.
- (4) (i) A is b- δw -closed.
(ii) A is δw_{gb} -closed and a $BC_{\delta w}$ -set. (iii)
 A is δw_{gb} -closed and λ - δw -b-closed.

Proof. The proof is similar to that of Lemma 2.16 and Theorem 2.28.

Remark 2.36. The following examples show that the concepts of

- (1) δw_{gs} -closed sets and $DC_{\delta w}$ -sets are independent of each other.
- (2) δw_{gs} -closed sets and λ - δw -semi-closed sets are independent of each other. (3)
- δw_{gp} -closed sets and $C_{\delta w}$ -sets are independent of each other.
- (4) δw_{gp} -closed sets and λ - δw -pre-closed sets are independent of each other. (5)
- δw_{gsp} -closed sets and $sFC_{\delta w}$ -sets are independent of each other.
- (6) δw_{gsp} -closed sets and λ - δw - β -closed sets are independent of each other. (7)
- δw_{gb} -closed sets and $BC_{\delta w}$ -sets are independent of each other.
- (8) δw_{gb} -closed sets and λ - δw -b-closed sets are independent of each other.

Example 2.37. (1) In Example 2.15, $\{a\}$ is a $DC_{\delta w}$ -set but not δw_{gs} -closed. (2) In Example 2.15, $\{a, c\}$ is δw_{gs} -closed but not a $DC_{\delta w}$ -set.

Example 2.38. (1) In Example 2.15, $\{a\}$ is λ - δw -semi-closed but not δw_{gs} -closed. (2) In Example 2.15, $\{a, c\}$ is δw_{gs} -closed but not λ - δw -semi-closed.

Example 2.39. Let $X=\{a, b, c\}$, $\tau=\{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $w=\{\emptyset, \{a\}, \{b, c\}, X\}$. Then

- (1) $\{b\}$ is a $C_{\delta w}$ -set but not δw_{gp} -closed.
- (2) $\{a, b\}$ is δw_{gp} -closed but not a $C_{\delta w}$ -set.

Example 2.40. (1) In Example 2.39, $\{b\}$ is λ - δw -pre-closed but not δw_{gp} -closed. (2) In Example 2.39, $\{a, b\}$ is δw_{gp} -closed but not λ - δw -pre-closed.

Example 2.41. (1) In Example 2.39, $\{b\}$ is a $sFC_{\delta w}$ -set but not δw_{gsp} -closed. (2) In Example 2.39, $\{a, b\}$ is δw_{gsp} -closed but not a $sFC_{\delta w}$ -set.

Example 2.42. (1) In Example 2.39, $\{b\}$ is λ - δw - β -closed but not δw_{gsp} -closed. (2) In Example 2.39, $\{a, b\}$ is δw_{gsp} -closed but not λ - δw - β -closed.

Example 2.43. (1) In Example 2.39, $\{b\}$ is a $BC_{\delta w}$ -set but not δw_{gb} -closed. (2) In Example 2.39, $\{a, b\}$ is δw_{gb} -closed but not a $BC_{\delta w}$ -set.

Example 2.44. (1) In Example 2.39, $\{b\}$ is λ - δw -b-closed but not δw_{gb} -closed.

(2) In Example 2.39, $\{a, b\}$ is δw_{gb} -closed but not λ - δw -b-closed.

Definition 2.45. Let w be a WS on a space X . A subset A of X is called

- (1) λ - δw - αg^* -closed if $A = L \cap F$, where L is a $\Lambda_{\alpha w}$ -set and F is δ -closed.
- (2) λ - δw - sg^* -closed if $A = L \cap F$, where L is a Λ_{sw} -set and F is δ -closed.
- (3) λ - δw - pg^* -closed if $A = L \cap F$, where L is a Λ_{pw} -set and F is δ -closed.
- (4) λ - δw - βg^* -closed if $A = L \cap F$, where L is a $\Lambda_{\beta w}$ -set and F is δ -closed.
- (5) λ - δw - bg^* -closed if $A = L \cap F$, where L is a Λ_{bw} -set and F is δ -closed.

Definition 2.46. Let w be a WS on a space X . A subset A of X is called

- (1) an δw -alc-set if $A = L \cap F$ where $L \in \alpha(w)$ and F is δw -closed.
- (2) an δw -slc-set if $A = L \cap F$ where $L \in \sigma(w)$ and F is δw -closed.

Lemma 2.47. Every $\Lambda_{\alpha w}$ -set (resp. Λ_{sw} -set, Λ_{pw} -set, $\Lambda_{\beta w}$ -set, Λ_{bw} -set) is λ - δw - αg^* -closed (resp. λ - δw - sg^* -closed, λ - δw - pg^* -closed, λ - δw - βg^* -closed, λ - δw - bg^* -closed).

Example 2.48. In Example 2.39, $\{c\}$ is λ - δw - αg^* -closed but not a $\Lambda_{\alpha w}$ -set.

Example 2.49. In Example 2.39, $\{b\}$ is λ - δw - sg^* -closed but not Λ_{sw} -set.

Example 2.50. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $w = \{\emptyset, X, \{b\}, \{b, c\}\}$. Then $\{a\}$ is λ - δw - pg^* -closed but not a Λ_{pw} -set.

Example 2.51. In Example 2.50, $\{c\}$ is λ - δw - βg^* -closed but not $\Lambda_{\beta w}$ -set.

Example 2.52. In Example 2.50, $\{a\}$ is λ - δw - bg^* -closed but not Λ_{bw} -set.

Lemma 2.53. For a WS w on a space X and a subset A of X , the following conditions are equivalent.

- (1) A is λ - δw - αg^* -closed.
- (2) $A = L \cap cl_{\delta}(A)$ where L is a $\Lambda_{\alpha w}$ -set.
- (3) $A = \Lambda_{\alpha w}(A) \cap cl_{\delta}(A)$.

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